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Comment

Comment on 'Full revivals in 2D quantum walks'

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Abstract

In the paper Štefaňák *et al* (2010 *Phys. Scr.* **T140** 014035) have proved that for any four-state quantum walk, there cannot be cycles longer than two steps. Our investigations reveal that they have not used the most general form of characteristic polynomials in their proof. Consequently, the result is not generally valid and hence there can be quantum walks having cycles longer than two steps.

(Some figures may appear in colour only in the online journal)

Štefaňák *et al* in [1] have shown that one can exploit the effect of localization to construct stationary solutions by using the Grover walk as an example. Moreover, they have found full revivals of a quantum state with a period of two steps and proved that there cannot be periods longer than two steps for any four-state quantum walk. They have mainly concentrated on the Grover walk which exhibits unusual effects of localization. In the momentum representation, the walk exhibits localization if the propagator of the quantum walk has eigenvalues which do not depend on the momenta. When the two different eigenvalues are independent of momenta, the linear combination of the corresponding eigenvectors results in an oscillatory behavior that shows full revivals of the quantum states.

Generally in quantum walks the state of the two particles after time t is obtained by applying the evolution operator U on the initial state of the walker. Initially, Fourier transform is carried out on the evolution operator and obtain the k, l momentum representation of U. The time evolution in the Fourier picture simplifies to

$$U_{k,l} = D(k, l) \,.\, C, \tag{1}$$

$$D(k, l) = D(k) \otimes D(l), \tag{2}$$

where

$$D(k, l) = \mathrm{Diag}(\mathrm{e}^{\mathrm{i}k}, \mathrm{e}^{-\mathrm{i}k}, \mathrm{e}^{\mathrm{i}l}, \mathrm{e}^{-\mathrm{i}l}). \tag{3}$$

As it is correctly mentioned in [1] if a quantum walk has a propagator which has two eigenvalues with a phase difference distinct from π or a propagator with more than two eigenvalues, full revivals will occur with periods longer than two steps. Later we will show that there are four-state quantum walks which satisfy these conditions and hence they can have periods longer than two steps.

In general when the propagator is in the form given in (1), C is an arbitrary unitary 4×4 matrix ($C = (c_{m,n}) \in \mathbb{R}^{4\times4}$) and λ is a constant eigenvalue then the characteristic polynomial of the propagator $U_{k,l}$ has the form

$$\begin{aligned}\lambda^4 &- f_1(c_{m,n}, \mathrm{e}^{\pm\mathrm{i}(k+l)}, \mathrm{e}^{\pm\mathrm{i}(k-l)})\lambda^3 + (f_2(c_{m,n}) \\ &+ f_3(c_{m,n}, \mathrm{e}^{\pm2\mathrm{i}k}, \mathrm{e}^{\pm2\mathrm{i}l}))\lambda^2 \\ &- f_4(c_{m,n}, \mathrm{e}^{\pm\mathrm{i}(k+l)}, \mathrm{e}^{\pm\mathrm{i}(k-l)})\lambda + \det C = 0.\end{aligned} \tag{4}$$

The explicit forms of f_1, f_2, f_3 and f_4 can be calculated when C is given. It is clear that the functions f_1, f_3 and f_4 generally depend on k, l and the coin operator while f_2 can depend on the coin operator alone. Therefore the coin elements $c_{m,n}$ are crucial in finding the solutions for λ.

Let us substitute the coin operator

$$C = C(\theta) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \sin\theta & 0 & 0 & -\cos\theta \\ \cos\theta & 0 & 0 & \sin\theta \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{5}$$

in (1) and the characteristic polynomial for $U_{k,l}$ becomes

$$\lambda^4 - 2\lambda^2\sin\theta + 1 = 0. \tag{6}$$

The solutions for (6) are of the form

$$\lambda_i(\theta) = \mathrm{e}^{\mathrm{i}\phi_i(\theta)} (i = 1, 2, 3, 4), \tag{7}$$

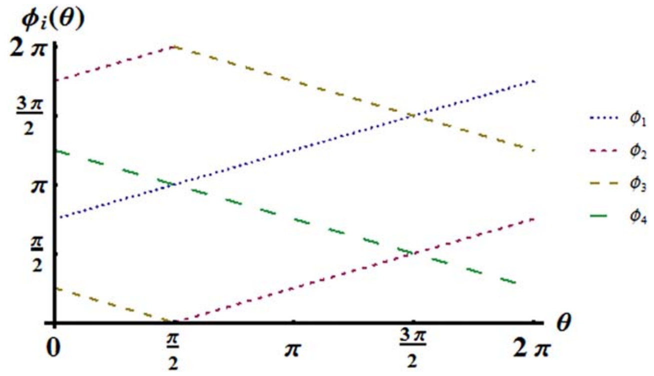

Figure 1. Phase angles ϕ_i ($\phi_i \to \phi_i$ mod 2π) of the solutions for the characteristic polynomial (6) as a function of θ.

where
$\phi_1(\theta) = \dfrac{3\pi}{4} + \frac{\theta}{2}$, $\phi_2(\theta) = \frac{3\pi}{4} + \frac{\theta}{2} + \pi$, $\phi_3(\theta) = \frac{\pi}{4} - \frac{\theta}{2}$, and $\phi_4(\theta) = \frac{\pi}{4} - \frac{\theta}{2} + \pi$.

According to (7) and figure 1 it is clear that unless $\theta = \frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$ this polynomial gives four distinct solutions. Due to $\sin\theta$ in the polynomial, solutions are complex. In other words (6) has real solutions only when $\theta = \frac{\pi}{2}$. It is clear from (4) the eigenvalues are dependent on the functions f_1, f_2, f_3 and f_4 and in turn they depend on the coin operator as well. Further it is important to note that all the eigenvalues in (7) are independent of quasimomentum.

Obviously the parameters in the coin operator play a major role in the quantum walk. For a certain class of coins, the characteristic polynomial will only give two real solutions while for other classes as stated above it will produce different complex solutions. The quantum walk stated above is a clear example for a walk which has a propagator with more than two eigenvalues and hence shows full revivals with periods longer than two.

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