Impact of Decoherence on Quantum Random Walks

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Abstract

This paper presents a qualitative study on decoherence in the context of two dimensional discrete-time quantum random walk (2D QRW) which is driven by a new global coin. A phase retarder has been introduced to different segments of the coin operator to generate decoherence and the variation in probability distribution has been analyzed in each circumstance. Two dimensional Hadamard walk (2D HW) has also been taken into consideration to make a comparison between decoherence produced by two coins. By controlling the characteristics of dephasing in the coin, one can tune the properties of quantum evolution and explore a great variety of scenarios of quantum random walk subjected to decoherence, such as transition from ballistic spread to diffusive spread, trapping and collapse of wave function. Further, it was found that when decoherence is introduced to quantum state of one walker in 2D HW with entangled initial coin states, decoherence was not observed in the state of the other walker. However, with the new 2D coin, states of the both walkers undergo decoherence when one of them experiences decoherence.

Keywords: **Decoherence, Two-dimensional discrete-time quantum random walk, Dephasing of coin operator, Trapping of wave function, Dynamical Collapse of wave function**

I. INTRODUCTION

Decoherence, from the perspective of a process that tends to reduce quantum coherence indicates that Quantum Random Walk may provide a fruitful testing ground for understanding the role played by decoherence in the transition from quantum to classical regime. Origin of the phenomenal study on the loss of quantum coherence in the context of Quantum Random Walk can be traced back to the very first practice of measurement based non-unitary evolution of quantum walk [1]. Later studies show that by dephasing the coin operator, one can transform quantum walk on a line into a classical random walk [2]. Studies on decoherence in one and higher dimensional quantum walks [3]-[9] have broaden the horizon of our comprehensibility in eliminating environmentally-induced noises and in using decoherence to tune the properties of quantum walks.

II. MODELING DECOHERENCE IN 1D QRW

Decoherence can be introduced to a 1D quantum dynamical system by dephasing the coin operator. This is done by attaching a phase retarder R to the coin as given in Eq. (1).

$$|\psi(t)\rangle = U_2^{t_2} U_1^{t_1} |\psi(0)\rangle \tag{1}$$

where $U_1 = S.(\mathbb{I}\otimes C)$, $U_2 = S.(\mathbb{I}\otimes R)(\mathbb{I}\otimes C)(\mathbb{I}\otimes R^{-1})$, $t = t_1 + t_2$, S and C are Shift and Coin operators respectively. During the first t_1 time steps walker follows a pure quantum walk. But the walker experiences the effect of decoherence during the rest of the time steps (t_2).

III. A NEW COIN OPERATOR

We have implemented a two dimensional discrete-time quantum random walk in which the motion of second walker depends on increment or decrement of the position of the first walker [10]. This is achieved by constructing a global unitary coin operator in the form

$$C_s = A_1 \otimes B_1 + A_2 \otimes B_2 \tag{2}$$

where $A_1 = \begin{pmatrix} \cos\alpha & 0 \\ \sin\alpha & 0 \end{pmatrix}$ $A_2 = \begin{pmatrix} 0 & \sin\alpha \\ 0 & -\cos\alpha \end{pmatrix}$ $B_1 = \begin{pmatrix} \cos\beta & \sin\beta \\ \sin\beta & -\cos\beta \end{pmatrix}$ $B_2 = \begin{pmatrix} \cos\phi & \sin\phi \\ \sin\phi & -\cos\phi \end{pmatrix}$ and $\alpha, \beta, \phi \,\epsilon[0,2\pi]$

We are particularly interested in the choice of $\alpha = 45^0$, $\beta = 45^0$ (Hadamard coin) and $\phi = 135^0$ as this combination yields fascinating results under decoherence.

IV. OUR WORK

We begin our analysis by introducing dephasing into one dimensional Hadamard walk. Phase retarder is defined as

$$R = e^{\frac{i\beta}{2}\sigma_z} = \begin{pmatrix} e^{\frac{i\beta}{2}} & 0 \\ 0 & e^{\frac{-i\beta}{2}} \end{pmatrix} \tag{3}$$

where $\beta \epsilon [0,2\pi)$ and $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is the Pauli spin matrix [2]. When no phase shift is introduced to the coin, Hadamard walk exhibits the typical ballistic spread in probability distribution for all initial conditions. But the distribution acquires a standard deviation comparable to that of the corresponding distribution of classical random walk when a phase retarder is embedded into the coin as given in Eq. (1). Further, averaging the probability values of the distribution over many trials makes the resulting distribution convergent to a classical binomial distribution rapidly.

Extending this idea into two-dimensional domain we have combined the phase retarder with different segments of our coin operator $(A_1, A_2, A_1 + A_2, B_1$ and $B_2)$ and have examined the distinctions among distributions. We have employed the techniques used in [8] to implement our two dimensional quantum walk numerically. Walkers are allowed to move on a fixed grid. Dynamical evolution of motion lasts for 50 time steps and decoherence is applied after 10th time step for the rest of the motion. Final distribution has been plotted after averaging the motion over 25 trials. All the Bell States along with the four product states have been taken into consideration when selecting initial conditions.

V. RESULTS AND DISCUSSION

As we observed, introduction of the phase retarder causes the motion of both walkers to become confined to the neighbourhood of their commencing positions as illustrated in FIG. 1-6. Further, we found that this behaviour has no dependence on where the retarder is placed (see the figures).

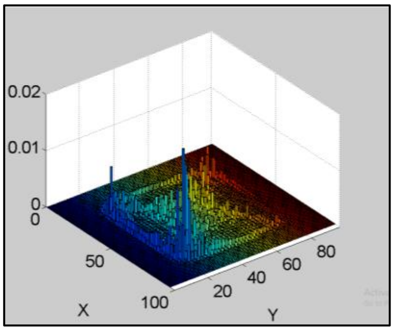

FIG. 1 : Probability distribution for ψ^+ state with no decoherence

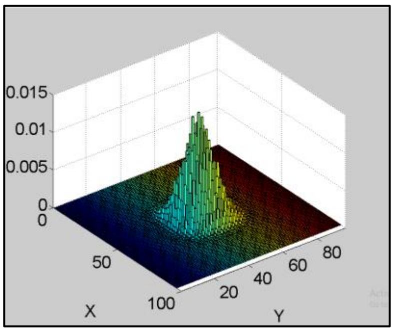

FIG. 2: Probability distribution for ψ^+ state when phase retarder is applied to $A_1 + A_2$

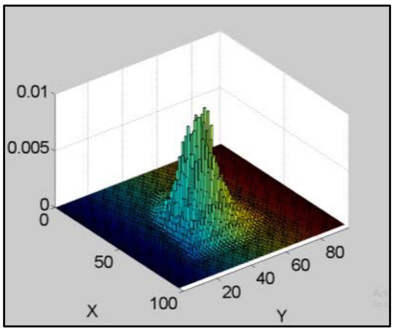

FIG. 3: Probability distribution for ψ^+ state when phase retarder is applied to B_1

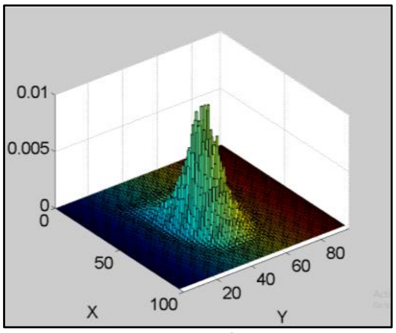

FIG. 4: Probability distribution for ψ^+ state when phase retarder is applied to B_2

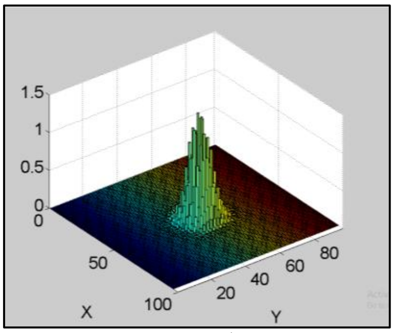

FIG. 5: Probability distribution for ψ^+ state when phase retarder is applied to A_1

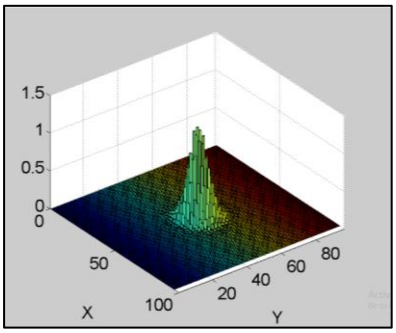

FIG. 6: Probability distribution for ψ^+ state when phase retarder is applied to A_2

Apparently, the area of confinement is independent from the initial condition but showcases sensitivity to the presence of the phase retarder in the coin operator. When phase retarder is applied either to matrix A_1 or A_2 of the coin operator, both walkers occupy a minimum area relative to other distributions in this study.

Moreover, unitary condition is well preserved while dephasing is functioning on $A_1 + A_2$, B_1 and B_2 individually. Yet, a non-unitary and collapse-like behaviour springs out when phase shifter is only attached to either matrix A_1 or A_2. Based on these observations one can safely state that upon the appearance of decoherence in the motion of the first or the second walker, the whole evolution gets trapped into a region about the starting point. This manifestation is the very antithesis of 2D HW in which the trapping is found along a single direction according to the presences of phase retarder in the coin. (FIG. 7and FIG. 8) In addition, it is not possible to achieve a non-unitary or apparent collapse-like behaviour by merely introducing segment-wise dephasing into 1D components of the 2D Hadamard coin operator.

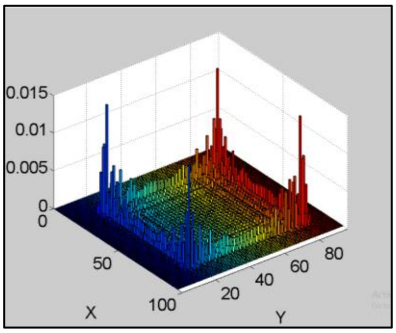

FIG. 7: Probability distribution of Hadamard walk for ψ^+ state with no decoherence

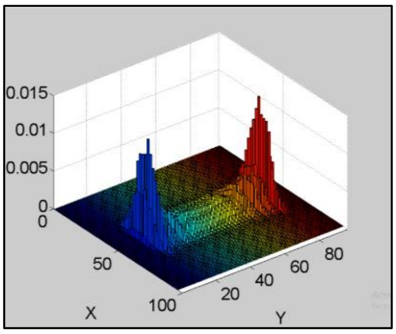

FIG. 8: Probability distribution of Hadamard walk for ψ^+ state when phase retarder is applied to first coin

Having been guided by the preceding work we have developed a discrete quantum-classical hybrid walk in which the motion of the first walker is governed by a classical coin and that of the second walker is governed by B_1 and B_2 conditional upon the coin toss of first walker. With an unbiased classical coin, a 'Bell Shaped' probability distribution appears in the motion of both walkers even though the second walker is controlled by a quantum coin operator. This unexpected peculiarity in the motion of the second walker can be explained by modifying Eq. (1). Defining the phase retarder in the following way,

$$R = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (4)$$

unitary operators of the motion can be expressed as

$$U_1' = S.(\mathbb{I} \otimes B_1) \text{ and}$$
$$U_2' = S.(\mathbb{I} \otimes B_2) = S.(\mathbb{I} \otimes R)(\mathbb{I} \otimes B_1)(\mathbb{I} \otimes R^{-1}) \quad (5)$$

With the help of above operators, initial state of the second walker can be evolved according to the outcome of the classical coin toss of the first walker. Due to this unique relationship between B_1 and B_2 via R in U_2', decoherence tends to appear in the second walker's motion and yields us with a 'Bell Shaped' probability distribution. Typical ballistic spread can be obtained in the second walker's motion whenever the walker moves in obedience to a single coin (B_1 or B_2) irrespective of the first coin toss.

VI. CONCLUTION

It is plausible to state that decoherence can be useful in controlling certain features of quantum random walk. On the other hand this ability can be utilized to gain a deep understanding of decoherence and how it functions on pure quantum systems leaving their evolution in peril. This paper has attempted to shed some light on the mysterious role played by decoherence in moving from quantum to classical regime which is yet to be revealed.

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